Errata for
Statistics of Bivariate von Mises Distributions

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February 21, 2015

Errata for K. V. Mardia and J. Frellsen. Statistics of bivariate von mises distributions. In T. Hamelryck, K. Mardia, and J. Ferkinghoff-Borg, editors, *Bayesian Methods in Structural Bioinformatics*, Statistics for Biology and Health, pages 159–178. Springer Berlin Heidelberg, 2012:

- In the last line of page 164 where the conditional distribution of ϕ given $\Psi = \psi$ for the *cosine model with positive interaction* is given, replace "$\mathrm{M}(\psi_\nu, \kappa_{13}(\psi))$" by "$\mathrm{M}(\mu - \psi_\nu, \kappa_{13}(\psi))$" — thanks to Luong Nguyen for pointing this out. See derivation in section 1.
- On page 170 equation (6.16) should be

$$C_{\mathrm{c}}^{-1} = \int_0^{2\pi} 2\pi I_0(\kappa_{13}(\psi)) \exp\{\kappa_2 \cos(\psi - \nu)\} \,\mathrm{d}\psi \,, \tag{1}$$

  thanks again to Luong Nguyen for pointing this out.

1 Conditional distribution for *cosine model with positive interaction*

From equation (6.12) in Mardia and Frellsen [2012] we can write the conditional distribution of ϕ given $\Psi = \psi$ for *cosine model with positive interaction* as

$$f_{\mathrm{c}+}(\phi|\psi) \propto \exp[\kappa_1 \cos(\phi - \mu) - \kappa_3 \cos(\phi - \mu - \psi + \nu)] \,. \tag{2}$$

According to the harmonic addition theorem (see e.g. Weisstein) we have

$$A_1 \cos(\omega t + \delta_1) + A_2 \cos(\omega t + \delta_2) = A \cos(\omega t + \delta) \tag{3}$$

where

$$A^2 = A_1^2 + A_2^2 + 2A_1A_2 \cos(\delta_1 - \delta_2) \tag{4}$$

$$\tan\delta = \frac{A_1 \sin(\delta_1) + A_2 \sin(\delta_2)}{A_1 \cos(\delta_1) + A_2 \cos(\delta_2)} \,. \tag{5}$$

Using the harmonic addition theorem we can rewrite equation (2) as

$$f_{c+}(\phi|\psi) \propto \exp[\kappa_{13}(\psi)\cos(\phi - \mu + \psi_\nu)] \tag{6}$$

where

$$\kappa_{13}^2(\psi) = \kappa_1^2 + \kappa_3^2 - 2\kappa_1\kappa_3\cos(\nu - \psi) \tag{7}$$

$$\tan\psi_\nu = \frac{\kappa_3\sin(\nu - \psi)}{\kappa_1 - \kappa_3\cos(\nu - \psi)}\,. \tag{8}$$

This means that conditional distribution of ϕ given $\Psi = \psi$ is von Mises distributed $\mathrm{M}(\mu - \psi_\nu, \kappa_{13}(\psi))$. As pointed out by Luong Nguyen the expression for the mean has a typo in Mardia and Frellsen [2012]. Also note that both mean and concentration in Mardia et al. [2007] are to be replaced by the expressions given here.

References

K. V. Mardia and J. Frellsen. Statistics of bivariate von mises distributions. In T. Hamelryck, K. Mardia, and J. Ferkinghoff-Borg, editors, *Bayesian Methods in Structural Bioinformatics*, Statistics for Biology and Health, pages 159–178. Springer Berlin Heidelberg, 2012.

K. V. Mardia, C. C. Taylor, and G. K. Subramaniam. Protein bioinformatics and mixtures of bivariate von mises distributions for angular data. *Biometrics*, 63(2):505–512, 2007.

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